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A Discrete Optimization Model for Designing Cellular Networks Considering Electromagnetic Radiation.

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ABSTRACT

The boom in demand for mobile phone throughout the world is causing a constant increase in the BST (Base Station Transmitter) number, particularly in Medan city. Such increase aims at achieve a better spatial frequency utilization to accomplish the growing number of users and services. However , the electromagnetic radiation (EMR) emitted from BTS are hazardous to human health. In this paper we address a linear mixed-integer programming model that combine the base station location problem, the frequency channel assignment problem and the base station connection to the fixed network considering the EMR. We solve the model using a feasible neighbourhood search approach.

Keywords

Cellular network; base station location; frequency channel assignment; network design; EMR; feasible search

1. Introduction

Mobile phone has become an icon for modern life of people around the world. It is not surprising that nowadays almost more than half of population of Indonesia own a mobile phone. The cellular system of mobile phone involves Base Transceiver Station (BST) antenna (Rappaport, 1996). The main purpose of such antenna is to provide good connection between mobile users and telephone network through radio waves in the coverage area. Therefore the boom in demand for this kind of modern communication systems led to the increase the number of BST. In Medan city, the capital of North Sumatra Province of Indonesia, for example, we can find more than 5000 BST antennas. The result The locations of these antenna could be anywhere, such as, residential complex, shopping mall, roof of houses and hospitals, etc.

The earliest cellular networks were comprised of a single base station with a large coverage area that was able to serve the entire mobile communication system. However, with the sharp increase of the demand of cellular services along with the limitations of the frequency spectrum use, a new structure of cellular networks developed. The first breakthrough was in deploying several base stations throughout the system area in order to decrease the coverage area of a base station and make frequency reuse possible.

Considering this new network configuration, new questions emerged. The first two questions were how many base stations should be deployed and where they should be placed. These questions have been addressed in the literature as the base station location problem (BSL) (Amaldi, Capone & Malucelli 2001a, Amaldi, Capone & Malucelli 2001b, Floriani 1998, Floriani & Mateus 1997, Galota, Glaßer, Reith & Vollmer 2001, Glaßer, Reith & Vollmer 2000, Mathar & Niessem 2000). From a mathematical programming perspective, the problem can be formulated as the problem of selecting from a group of candidate base stations a subgroup of minimum cost that is able to cover the entire system area. Actually, this formulation is a particular instance of the well known facility location problem, where the facilities are the base stations. The facility location problem is NP-hard as shown by Mirchandani (Mirchandani & Francis 1990). Therefore, the BSL is also NP-hard.

The second question to emerge was how to maximize the frequency reuse keeping at the same time the frequency interference under a threshold. Considering the second generation (2G) cellular networks as the GSM networks, which is based on Frequency Division Multiple Access (FDMA) and Time Division Multiple Access (TDMA), this question has been addressed in the literature as the frequency channel assignment (FCA) problem (Ardal, Hipolito, van Hoesel & Jansen 1995, Bornd¨orfer, Eisenbl¨atter, Gr¨otschel & Martin 1997, Bornd¨orfer, Eisenbl¨atter, Gr¨otschel & Martin 1998, Castelino, Hurley & Thiel 1996, Fischetti, Lepschy, Minerva & Romanin-Jacur 2000, Floriani 1998, Floriani & Mateus 1997, Hale 1980, Hurkens & Tiourine 1995, Hurley, Smith & Thiel 1995, Hurley, Thiel & Smith 1996, Jaumard & Vovor 1998, Mateus & Loureiro 1998, Web n.d., Rappaport 1996, Schneider 1997, Smith, Hurley & Thiel 1998, Smith, Allen, Hurley & Watkins 1998, Smith & Hurley 1997, Tiourine, Hurkens & Lenstra 1995). Again, from a mathematical programming viewpoint, the problem can be formulated as the problem of assigning frequency channels to the selected base stations subject to demand and frequency channel interference constraints. The objective function can vary according to the problem context. When the assignment is based on variable-size frequency spectrum, the objective usually intends to minimize the number of frequency channels used (Ardal et al. 1995, Hale 1980, Hurkens & Tiourine 1995, Hurley et al. 1995, Hurley et al. 1996, Smith, Allen, Hurley & Watkins 1998, Smith, Hurley & Thiel 1998, Tiourine et al. 1995). On the other hand, when the frequency spectrum size is fixed, the objective seeks to minimize the interference among frequency channels (Bornd¨orfer et al. 1997, Bornd¨orfer et al. 1998, Castelino et al. 1996, Hurkens & Tiourine 1995, Schneider 1997, Smith, Allen, Hurley & Watkins 1998, Tiourine et al. 1995) or minimize the number of channels of unserved demand (Fischetti et al. 2000, Jaumard & Vovor 1998). Hale (Hale 1980)shows that the FCA problem is NP-hard exploiting it's equivalency to the graph coloring problem.

For the third generation (3G) cellular networks as the Universal Mobile Telecommunication System (UMTS) networks, the medium access is based on the Wideband Code Division Multiple Access (WCDMA) technology rather than on TDMA or FDMA. In the

CDMA technology, the whole spectrum is used for each connection rather than frequency channels and time slots. Despite the medium access technology, the problem of frequency spectrum reuse still exists in 3G networks since the frequency spectrum allocated is finite though it is wider than the one available for the second generation networks. Therefore, the interference problem also shows up in 3G networks, but now it is addressed by control power mechanisms (Amaldi et al. 2001a, Galota et al. 2001, Glaßer et al. 2000).

The third question emerging from this new network configuration was how to connect the candidate base stations to the fixed telephone network. This question has been addressed in the literature as the topological network design (TND) problem (DeSousa, Formigoni, Bergamaschi, Branquinho, Yamamoto, Machado & Filho 2000, Dutta & Kubat 1999, Kubat & Smith 2000). Again according to the mathematical programming viewpoint, this problem can be formulated as the problem of defining the network topology of minimum cost that is able to connect the candidate base stations to the fixed telephone network. Some of the features that can be considered in the problem formulation are the use of hubs, different kinds of media, and link capacities. The topological network design problem is a classical one that has already been explored in other network contexts. Dutta (Dutta & Kubat 1999) shows that the TND problem is NP-hard.

As introduced above, the cellular network design (CND) is a complex problem comprehending the base station location (BSL) problem, the frequency channel assignment (FCA) problem and the topological network design (TND) problem. Therefore, the CND is an NP-hard problem since the BSL, FCA and TND problems are all NP-hard. It is easy to notice that you can always reduce the CND to any one of the BSL, FCA or TND problems. This is enough to prove the NP-hardness of the CND problem.

Numerous mathematical programming formulations have been proposed to solve separately each one of these problems, as well as corresponding solution techniques. However, it seems intuitive that both the FCA and the TND problems depend on the BSL problem solution, as well as their solutions. Previous computational analyses reported in (Mazzini & Mateus 2001) showed that there is a tradeoff between the BSL and FCA and between the BSL and TND problems. In order to show these relations, the authors solved the CND problem for a small instance. The problem was solved both separately and integratedly. In the first approach, the BSL problem was solved, and its solution was used as an instance for the FCA problem. In the BSL problem solution, 15 base stations were deployed. The FCA solution showed that this number was underestimated since 15% of the whole frequency channel demand was unserved. In the second approach, 16 base stations were deployed and there was no unserved frequency channel in the system. This computational result showed that the minimum BSL cost network was not able to serve the network demand. The paper also called the attention for the fact that although there were some base stations with available capacity in the first approach solution, it could not be used due to frequency channel interference. Considering the BSL and TND, the experiments showed that the set of base stations deployed was different for each approach and that the solution cost was higher for the first approach than for the second one. This result showed that the integrated approach was more cost-effective compared to the separate approach. A practical evidence that it is more cost-effective to address the relations among the subproblems is that the cellular network companies solve them iteratively. They pursue the minimum network cost but also the quality of service. For instance, if the BSL solution is not able to provide quality of service, it is solved again with some extra constraints to force a capacity increase. However, it is more effective to address the problem tradeoff by an integrated formulation than by an iterative approach. Following this recent trend, research efforts have also been proposed (Mazzini & Mateus 2001, Mazzini, Mateus & Luna 2001, Mathar & Schmeink 2000) where the BSL and the FCA problems have been integrated into the same formulation.

An integrated approach to solve the cellular network design leads to a large scale linear mixed integer programming model with millions of variables and constraints. Unless the mathematical formulation has special structure, it appears to be unlikely that an exact solution can be obtained. Therefore, a typical approach is to look for solution techniques to provide effective bounds for the optimal solution. Considering that the cellular network design is a problem of minimizing costs, techniques such as linear relaxation (Nemhauser & Wosley 1988), Lagrangean relaxation (Nemhauser & Wosley 1988, Fisher, Northup & Shapiro 1975, Fisher 1981, Fisher 1985, Geoffrion 1974, Geoffrion 1978, Reeves 1993, Shapiro 1979), Benders decomposition (Bazaraa, Jarvis & Sherali 1990, Lasdon 1971) as well as cutting plane methods (Ferreira & Wakabayashi 1996) can be used to provide good lower bounds to the problem. On the other hand, heuristics (Reeves 1993) such as simulated annealing, tabu search, genetic algorithms as well as specific problem-based heuristics can be used to provide feasible solutions as well as upper bounds for the general problem. These bounds achieved by these latter methods can still be improved through tree search algorithms such as branch and bound.

This paper presents a solution algorithm to solve the cellular network design problem (Mazzini & Mateus 2001, Mazzini et al. 2001) for second generation networks. The problem is modeled as a linear mixed integer programming model that integrates the base station location (BSL), the frequency channel assignment (FCA) and the topological network design (TND) problems. The solution algorithm is based on a Lagrangean relaxation technique and subgradient method and consists in evaluating lower and upper bounds for the problem. The lower bound is derived by the solution of the Lagrangean dual problem and the upper bounds are computed based upon the Lagrangean problem solutions through a Lagrangean heuristic. These bounds are further improved through a customized branch and bound algorithm.

2. Mathematical modeling

The basic framework of the model to be formulated for cellular topological network design considering power density (CTND-PD) in this paper is adopted from Mazzini et al. (2003). Nevertheless, they only integrated BST location, The frequency channel assignment (FCA) and the network design in their model.

Firstly we define the notation used.

Set

I : set of candidate base transceiver stations (BTSs)

J : set of hubs

K : set of switches

L : set of points of demand

M : set of frequency channels (FCs)

N^i : set of interfering BTSs to BTS i $\quad \forall\, i \in I$

Parameters

cost

α_i	cost of installing the BTS i	$\forall\, i \in I$
β_{ij}	cost to connect the BTS i to the hub j	$\forall\, i \in I, \forall\, j \in J$
γ_{ik}	cost to connect the BTS i to the switch k	$\forall\, i \in I, \forall\, k \in K$
δ_{jk}	cost to connect the hub j to the switch k	$\forall\, j \in J, \forall\, k \in K$
$\eta_{ii'}$	cost for removing BTS i to i’	$\forall\, i \in I, \forall\, i' \in I$

Other parameters

κ_i	the maximum number of points that can be served by BS I	$\forall\, i \in I$
λ_i	the maximum number of FCs that can be assigned to BS I	$\forall\, i \in I$
ρ_j	the demand in number of communication channels of point l	$\forall\, l \in L$
σ	the number of communication channels carried by a FC	
d	the minimum orthogonal frequency distance between adjacent FCs	
ρ	power density	

$$\mu_{ij} = \begin{cases} 1 & \text{if BS } i \text{ covers point } j \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, i \in I, \forall\, l \in L$$

and the binary variables are defined as:

$$y_i = \begin{cases} 1 & \text{if the BTS } i \text{ is selected} \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, i \in I$$

$$q_{ij} = \begin{cases} 1 & \text{if the BTS } i \text{ is connected to the hub j} \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, i \in I, \forall\, j \in J$$

$$r_{ik} = \begin{cases} 1 & \text{if the BTS } i \text{ is connected to the hub j} \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, i \in I, \forall\, s \in S$$

$$u_{jk} = \begin{cases} 1 & \text{if the BTS j is connected to the hub k} \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, j \in J, \forall\, k \in K$$

$$v_{im} = \begin{cases} 1 & \text{if the FC m is assigned to BS } i \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, i \in I, \forall\, m \in M$$

Continuous variables

x_{il} the rate of point l served by BTS i $\qquad \forall\, i \in I, \forall\, l \in L$

The values of x_{il} are between 0 and 1

Objective function

In general, the objective of a network distribution problem is to minimize cost. Therefore, it would be the same for the case of the cellular topology network design considering power density (CTND-PD). The objective consists of the costs of installing BTS, the costs to connect the located BTS to the fixed telephone network and the costs for changing location of BTS due to the unfulfillment of power density threshold.

The costs of locating BTS can be formulated as : $C_1 = \sum_{i \in I} \alpha_i y_i$

Connection costs for each BTS selected to the hub can be written as : $C_2 = \sum_{i \in I} \sum_{j \in J} \beta_{ij} q_{ij}$

Connection costs for each BTS selected to the switch :$C_3 = \sum_{i \in I} \sum_{k \in K} \gamma_{ik} r_{ik}$

Connection costs from hub to the switch: $C_4 = \sum_{j \in J} \sum_{k \in K} \delta_{jk} u_{jk}$

Removing cost of BTS from i to i': $C_5 = \sum_{i \in I} \sum_{i' \in I} \eta_{ii'} y_{ii'}$

The objective of the problem can be expressed as to minimize costs

$$Min\ z = C_1 + C_2 + C_3 + C_4 + C_5 \qquad (1)$$

Constraints

There are several constraints needed to be imposed in order to guarantee the feasibility of the BSTL, FCA and power density based cellular network design (PD-CND).

Firstly, we formulate the BSTL problem. As in the usual location problem it is necessary to guarantee that every point of demand is served by the antenna of BST. Furthermore , in the model we should guarantee that a candidate base station which is not selected must not serve any point of demand, and there must be an upper bound for the number of points to be served by a selected base station. This condition is presented in Equations (2) and (3)

$$\sum_{i\in I} \mu_{il} x_{il} \geq 1, \qquad \forall l \in L \tag{2}$$

$$\sum_{l\in L} \mu_{il} x_{il} \leq \kappa_i y_i, \ \forall i \in I \tag{3}$$

The set *L* of points of demand represents the service area of the cellular network. The parameters μ_{il} represent the coverage area of each base station. Finally, the linear variables x_{il} define how much of a point *l* is served by a base station *i*. If $x_{il} = 1$, the base station *i* serves completely (100%) the demand of the the point *l*. If $0 \leq x_{il} \leq 1$, the base station *i* serves only part of the demand of the point *l*. In this case, constraint (2) guarantees that another or other base stations serve the remaining demand not served by i. If $x_{il} = 0$, the point *l* is not served by the base station *i*. The idea to make x_{il} linear is to allow that a point *l* can be served by more than one base station.

Secondly for the CND problem. In order to guarantee the feasibility of the CND problem the model has to make sure that all selected base stations and the fixed network are well connected. the set of constraints (4) guarantees for all base stations that if a base station i is selected, it must be connected at least to a hub or switch, while the set of constraints (5) guarantees for all hubs that if there is any selected base station connected to a hub *j*, it must be connected at least to a switch.

$$\sum_{j\in J} q_{ij} + \sum_{k\in K} r_{ik} \geq y_i, \ \forall i \in I \tag{4}$$

$$\sum_{k\in K} u_{jk} \geq q_{ij}, \ \forall i \in I, \forall j \in J \tag{5}$$

In the FCA problem, the feasibility depends on channel interference, demand and capacity constraints. The sets of FCA constraints are represented by equations (6) through (9). The set of constraints (6) guarantees that adjacent frequency channels assigned to the same base station are separated from each other by at least *d* orthogonal frequency channels. The set of constraints (7) guarantees that if a frequency channel *m* is assigned to a base station *i*, it must not be assigned to any one of the base stations belonging to the interfering set N^i. The interfering sets

N^i define for each base station i the other base stations that cause interference to its frequency channels. The set of constraints (8) imposes that the number of frequency channels assigned to a base station *i* is enough to serve the demand of the points served by that base station. The parameter *σ* defines how many communication channels can be carried by a frequency channel. In case of FDMA technologies it is set to 1, while it can be set to 3 in case of some TDMA technologies. Finally, the set of constraints (9) guarantees that a frequency channel can be assigned to a base station *i* only if the base station is selected, and it still imposes a bound on the maximum number of frequency channels assigned to a base station *i*.

$$\sum_{m=1}^{g+d} v_{im} \leq 1,\ \forall i \in I, \forall g \in \{1,\dots,|M|-d\} \tag{6}$$

$$v_{im} + \sum_{g \in N^i} v_{gm} \leq 1,\ \forall i \in I,\ \forall m \in M \tag{7}$$

$$\sigma \sum_{m \in M} v_{im} \geq \sum_{l \in L} \rho_l x_{ij},\ \forall i \in I \tag{8}$$

$$\sum_{m \in M} v_{im} \leq \lambda_i y_i,\ \forall i \in I \tag{9}$$

Constraints (10) and (11) express the binary and continuous variables, respectively.

$$y_i, \mathrm{v}_{im}, \mathrm{q}_{ij}, \mathrm{r}_{ik}, \mathrm{u}_{jk} \in \{0,1\},\ \forall i \in I, \forall m \in M, \forall j \in J, \forall k \in K \tag{10}$$

$$0 \leq x_{il} \leq 1,\ \forall i \in I, \forall l \in L, \tag{11}$$

3. The Algorithm

We adopt the approach of examining a reduced problem in which most of the integer variables are held constant and only a small subset allowed varying in discrete steps.

The steps of the procedure can be summarized as follows.

Step 1. Solve the problem ignoring integrality requirements.

Step 2. Obtain a (sub-optimal) integer-feasible solution, using heuristic rounding of the continuous solution.

Step 3. Divide the set I of integer variables into the set I_1, at their bounds that were nonbasic at the continuous solution, and the set I_2, $I = I_1 + I_2$.

Step 4. Perform a search on the objective function, maintaining the variables in I_1 nonbasic and allowing only discrete changes in the values of the variables in I_2.

Step 5. At the solution in step 4, examine the reduced costs of the variables in I_1. If any should be released from their bounds, add them to the set I_2 and repeat from step 4, otherwise terminate.

It should be noted that the above procedure provides a framework for the development of specific strategies for particular classes of problems.

The integer results are kept in superbasic variables set. Then we conduct an integer line search to improve the integer feasible solution (Mawengkang et al., 2012)

4. Conclusions

In this article, a solution approach to the NP-hard large scale cellular network design problem is proposed. The problem is formulated as a comprehensive linear mixed integer programming model including the base station location, the frequency channel assignment and the topological network design problems.

We propose a feasible neighbourhood search approach for solving the model.

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