Designing a Learning Media using Matlab for Matrix and Its Operations

Binur Panjaitan

Mathematics Education Department, Nommensen HKBP University Medan Jl. Sutomo No. 4A Medan, North Sumatera, Indonesia.

Abstract

The problem in this research is to utilize known methods to solve problems in algebra matrix operations using MatLab. The purpose of this study is to design a learning media using matlab. The research instrument used in this paper is a check list to check compliance with the calculated results of the program directly. After testing the program for $m \times n$ matrix operations easier and shorter using matrix operations program m-file.

Keywords : Learning media, matlab, matrix

INTRODUCTION

Learning is a complex process that occurs in every person throughout his life. The learning process was due to the interaction between man and his environment. Therefore, learning can occur anytime and anywhere. One sign that a person has to learn is their behavioral changes in that person. Such amendment may be changes in the level of knowledge, skill or attitude.

The advancement of technology during the past 10 years has greatly affected how we teach, what we teach and how our students learn in our undergraduate courses. Computer technology is greatly in^ouencing the way we do mathematics and the problems which are accessable to our students. Before the introduction of Mathematica, Maple and other computer algebra systems, algebraic manipulation discouraged many students from studying mathematics and it is this reason that makes students think that mathematics is the most di±cult subject. This dillusion keeps students from appreciating mathematics as a subject full of power and ideas and that it requires a lot of critical and logical thinking. By using computer algebra systems like Mathematica, Maple and Derive we can spend less time on doing algebraic

manipulation, routine problem solving and sketching useless graphs, and spend more time on understanding concepts, and applying them to solve more realistic and challenging problems. The numerical and graphical capabilities of these software make students see what they could never before even imagine. These powerful capabilities can also help students make discoveries with our careful guidance.

In the development of science and technology, the computer appears as a very significant tool in saving time and energy to solve various problems. Computer as a tool capable of solving problems in data processing speed and exactitude. Computers are one of the electronic devices that are essential in applied technology-based learning in the classroom, so that students will be familiar to technological developments. Therefore, the author would like to take advantage of the mathematical theory and the computer tools to solve the problems of the matrix. One of software that can be used to solve the matrix problem is matlab.

Matlab is widely used in all areas of applied mathematics, in education and research at universities, and in the industry. MATLAB stands for MATrix LABoratory and the software is built up around vectors and matrices. This makes the software particularly useful for linear algebra but MATLAB is also a great tool for solving algebraic and differential equations and for numerical integration. MATLAB has powerful graphic tools and can produce nice pictures in both 2D and 3D. It is also a programming language, and is one of the easiest programming languages for writing mathematical programs. MATLAB also has some tool boxes useful for signal processing, image processing, optimization, etc.

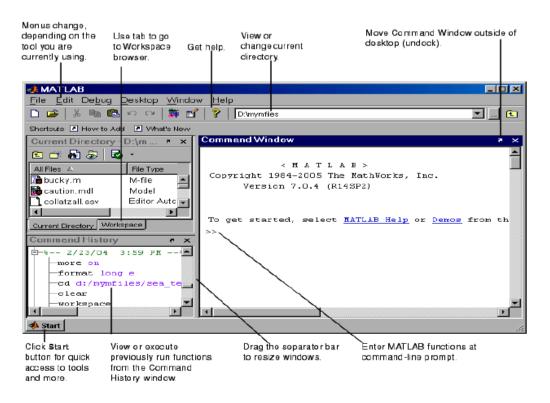


Figure 1. The graphical interface to the MATLAB workspace

Matlab was first conceived by Cleve Moler who wrote this language using FORTRAN (Translation Format) in Mexico University and Stanford University in the late of 1970s. The first edition of matlab is written by Steve Bangert where he wrote the part of interpreter while the graph is written by Steve Kleiman. John Little and Cleave Moler write the analytical Routines, user instructions (User's Guide) and most of the M-Files. Since the first part matlab written, a lot of people who join the development team matlab and successfully delivered or a substantial contribution. Now matlab capabilities even further beyond its original capabilities.

In computer science, matlab is defined as a language with a (high - performance) for high performance technical computing problems. Matlab is able to integrate computing, visualization, and programming in a model that is very easy to use where problems and solutions are expressed in familiar mathematical notation.

LEARNING MEDIA

Media education in general, is a teaching and learning tool. Everything that can be used to stimulate thoughts, feelings, concerns and abilities or skills of learners so as to facilitate the process of learning. This limitation is quite broad and includes in-depth understanding of the source, the environment, human beings and the method used for the purpose of learning / training. Meanwhile, according to Briggs (1977) is a physical means of learning media to deliver content / learning materials such as books, movies, videos and so on. Then, according to the National Education Associaton (1969) revealed that learning is a means of communication media in print and point of view heard, including technology hardware.

Media can be defined by their technology, their symbol systems, and their processing capabilities. The most obvious characteristic of a medium is its technology, the mechanical and electronic aspects that determine its function and to some extent its shape and other physical features. These are the characteristics that are commonly used to classify a medium as a "television," a "radio," and so on. The cognitive effects of these characteristics, if any, are usually indirect. Characteristics such as size, shape, and weight makes it more likely that a student will learn with a book while on a bus but not a computer, though of course this is changing as computers get smaller, lighter, and cheaper. On the other hand, some cognitive effects of technology are more direct. For example, the size and resolution of many computer screens is such that reading text may be more difficult than it is with books (Haas, 1989).

In the past century, various forms of media have been used to convey instruction and to support learning. Examples of instructional media include traditional means of delivering instruction (chalkboards, textbooks, overhead projectors, and teachers), mass media used for education (newspapers, movies, radio, and television), and the newer "electronic" instructional media (computers, interactive video, and multimedia systems). All instruction requires the selection and use of at least one medium to deliver instruction. Many alternative media and mixtures of media may be chosen for any given learning goal and group of students. Thus, research questions have compared the learning benefits of various media and mixes of media for different types of learning goals and students at different ages and aptitude levels. Thousands of studies have been and continue to be conducted.

There are several types of learning media such as:

- a. Visual Media: graphs, diagrams, charts, charts, posters, cartoons, comics.
- b. Audial media: radio, tape recorders, language labs, and the like,
- c. Projected still media: slides; over head projector (OHP), in focus and the like,
- d. Projected motion media: movies, television, video (VCD, DVD, VTR), computers and the like.

Technology such as computers can be used as props in the learning of mathematics. Computers can help educators to explain to students about math, especially those related to technology and its application in life. The use of technology also make learning more fun. With this technology media, communication and interaction between educators with learners become more effective and efficient.

The utilization of electronic technology in learning provide reinforcement to the learning paradigm shift patterns. The use of information technology and multimedia is one way of effective and efficient in conveying information to students. Exercises and experiments can be conducted exploratory mathematics learners using simple programs for the establishment and strengthening of the concept, create mathematical modeling, and develop strategies to solve problems. There are several forms of computer-assisted learning interactions, namely the form of training and practice, tutorials, games, simulations, interactive discovery, presentation or demonstration, communications tests, resources, and problem solving as well as various other forms of interaction.

Salomon (1974, 1979) suggests that these characteristics should be used to define, distinguish, and analyze media since they are relevant to the way learners represent and process information from a medium. He contends that certain symbol systems may be better at representing certain tasks and that information presented in different symbol systems may be represented differently in memory and may require different mental skills to process. The research reviewed here supports and elaborates on this contention. For example, studies will be examined that illustrate how symbol systems characteristic of certain media can connect mental representations to the real world in a way that learners with little prior knowledge have trouble doing on their own, without the representation of information in these symbol systems.

MATRIX AND ITS OPERATIONS

Matrix is a set of scalar arranged in rows and columns in rectangular form. Scalar in the matrix is said to be the element of the matrix. Matrix size indicated by the number of rows \times columns. The specifics of symbolic matrix notation vary widely, with some prevailing trends. Matrices are usually symbolized using upper-case letters, while the corresponding lower-case letters, with two subscript indices represent the entries. For example, a_{21} represents the element at the second row and first column of a matrix A. In addition to using upper-case letters to symbolize matrices, many authors use a

special typographical style, commonly boldface upright (non-italic), to further distinguish matrices from other mathematical objects.

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$
(1)

BASIC OPERATION

There are several of basic operations that can be applied to matrices, called matrix addition, matrix subtraction, scalar multiplication, transposition, matrix multiplication, row operations (Brown, 1991).

a. Matrices addition

Two matrices can be added if both of them have the same size. Suppose that $A = [a_{ij}]$ and $B = [b_{ij}]$ are the matrices of size $m \times n$. The sum A + B of two $m \times n$ matrices A and B is calculated entrywise:

$$(A+B)_{ij} = A_{ij} + B_{ij}, \text{ where } 1 \le i \le m \text{ and } 1 \le j \le n$$

$$(2)$$

Here are the properties of matrix addition:

Suppose that A, B, C are the matrices of size $m \times n$, then : (i). A + B = B + A (*Commutative*) (ii). (A + B) + C = A + (B + C) (Associative)

b. Matrices substraction

If A and B have the same number of rows and columns, then: A - B is defined as A + (-B).

c. Matrice multiplication

1) Scalar multiplication

Suppose c is a scalar. The product cA of a scalar c and a matrix A is computed by multiplying every entry of A by c.

$$(cA)_{ij} = c \cdot A_{ij}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$$cA = \begin{bmatrix} ca_{11} & ca_{12} & \cdots & ca_{1n} \\ ca_{21} & ca_{22} & \cdots & ca_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ ca_{m1} & ca_{m2} & \cdots & ca_{mn} \end{bmatrix}$$

2) Multiplication of two matrices

Two matrices can be multiplied if the matrices have same size. If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then their matrix product AB is the $m \times p$ matrix whose entries are given by dot product of the corresponding row of A and the corresponding column of B as eq. (3) show:

$$[AB]_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{in}b_{nj} = \sum_{r=1}^{n} a_{ir}b_{rj}$$
(3)

The properties of multiplication of two matrices is:

- a. Matrix multiplication satisfies the rules (AB)C = A(BC) (associative)
- b. It's also satisfie that (A + B)C = AC + BC as well as C(A + B) = CA + CB (*left and right distributive*)
- c. AB Matrix multiplication is not commutative. It means that $AB \neq BA$.

TRANSPOSITION OF A MATRIX

The transpose of an $m \times n$ matrix A is the $n \times m$ matrix A^T formed by turning rows into columns and vice versa:

$$(A^T)_{ij} = A_{ji}$$

Matrix A is called orthogonal if $AA^T = A^T A = I_n$.

DETERMINANT OF A MATRIX

The determinant det(A) or |A| of a square matrix A is a number encoding certain properties of the matrix. A matrix is invertible if and only if its determinant is nonzero. Its absolute value equals the area (in R^2) or volume (in R^3) of the image of the unit square (or cube), while its sign corresponds to the orientation of the corresponding linear map: the determinant is positive if and only if the orientation is preserved. det A atau |A|

The determinant of 2×2 is given by:

$$\det \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

The determinant of 3-by-3 matrices involves 6 terms (rule of Sarrus). The more lengthy Leibniz formula generalises these two formulae to all dimensions (Bronson, 1989).

Adding a multiple of any row to another row, or a multiple of any column to another column, does not change the determinant. Interchanging two rows or two columns affects the determinant by multiplying it by -1 (Brown, 1991). Using these operations, any matrix can be transformed to a lower (or upper) triangular matrix, and for such matrices the determinant equals the product of the entries on the main diagonal; this provides a method to calculate the determinant of any matrix. Finally, the Laplace expansion expresses the determinant in terms of minors, that is, determinants of smaller matrices (Mirsky, 1990). This expansion can be used for a

recursive definition of determinants that can be seen to be equivalent to the Leibniz formula.

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Determinants can also be used to solve linear systems using Cramer's rule, where the division of the determinants of two related square matrices equates to the value of each of the system's variables (Brown, 1991).

INVERTIBLE MATRIX AND ITS INVERS

A square matrix A is called invertible or non-singular if there exists a matrix B such that

$$AB = BA = I_n$$

If B exists, it is unique and is called the inverse matrix of A, denoted A^{-1} . Matrix inversion is the process of finding the matrix B that satisfies the prior equation for a given invertible matrix A.

A square matrix that is not invertible is called singular or degenerate. A square matrix is singular if and only if its determinant is 0. Singular matrices are rare in the sense that a square matrix randomly selected from a continuous uniform distribution on its entries will almost never be singular.

Non-square matrices (m-by-n matrices for which $m \neq n$) do not have an inverse. However, in some cases such a matrix may have a left inverse or right inverse. If A is m-by-n and the rank of A is equal to n, then A has a left inverse: an $n \times m$ matrix B such that BA = I. If A has rank m, then it has a right inverse: an $n \times m$ matrix B such that AB = I.

GENERATING M-FILE

First step, the author make some function based on the algorithm of matrice operation. There are several function in main program of the m-file. The main program is saved as matrix.m in the directory of m-file project.

One of the body function was written as below:

```
function C = MatMatDot(A, B)
% C = MatMatDot(A, B)
% This computes the matrix-matrix product C =A*B
(via dot products) where
% A is an m-by-p matrix, B is a p-by-n matrix.
[m,p] = size(A);
[p,n] = size(B);
C = zeros(m, n);
for j=1:n
     % Compute j-th column of C.
    for i=1:m
         C(i,j) = A(i,:) * B(:,j);
    end
end
function y = MatVecR0(A, x)
% y = MatVecRO(A, x)
% Computes the matrix-vector product y = A^*x
%(via saxpys) where A is an m-by-n matrix and x
% is a columnn-vector.
[m,n] = size(A);
y = zeros(m, 1);
    for i=1:m
        y(i) = A(i,:) *x;
end
```

Figure 2. Code function for multiplication of matrices

RESULT AND PROGRAM TEST

PROGRAM INTERFACE :

Open matrix.m file in Matlab directory and then select 'run' to run the program. After that the program will show as follow:

Figure 3. Main menu of the matrix program

From the display of the program interface above, the user is asked to enter a number between 1 to 7 based on the option above. If you enter a number other than the desired number then there will be a warning message such as "your input is wrong...!!!". For example, the user input 3, then it will show as below:

```
****
# Multiplication of Matrices
*****
Please input the number of row and column of matrix A
%input the number of row of matrix A, THEN press ENTER
   Number of row (m) = 2
%input the number of column of matrix A, THEN press ENTER
   Number of column (n) = 3
%input the matrix A
   A = [2 \ 1 \ 2; \ 1 \ 2 \ 1]
   A =
       2 1 2
       1 2 1
Please input the number of row and column of matrix B
%input the number of row of matrix B, THEN press ENTER
   Number of row (m) = 3
%input the number of column of matrix B, THEN press ENTER
   Number of column (n) = 3
%input the matrix B
         B = [2 \ 1 \ 4; \ 1 \ 2 \ 3; \ 2 \ 2 \ 1]
         в =
             2 1 4
             1 2 3
             2 2 1
%press enter to show the result
Matrix A * Matrix B = Matrix C
         C =
             9 8 13
             6 7 11
***
```

Figure 4. Program interface for multiplication of matrices

Another example, if the user input 6 as the choice, then it will show:

```
******
# Inverse of a Matrix
***
Please input the number of row and column of matrix A
%input the number of row of matrix A, THEN press ENTER
   Number of row (m) = 3
%input the number of column of matrix A, THEN press ENTER
   Number of column (n) = 3
%input the matrix A
   A = [1 \ 3 \ 3; \ 1 \ 4 \ 3; \ 1 \ 3 \ 4]
   A =
      1 3 3
      1 4 3
        3
      1
          4
%press enter to show the result
  Inv(A) =
         7 -3 -3
         -1 1 0
         -1
            0
               1
******
```

Figure 5. Program interface for inverse of a matrices

CONCLUSION

Based on the description above, the authors draw some conclusions as follows:

- 1. Computers are very useful as a learning media for students, especially in solving a difficult problems.
- 2. Matlab program is very helpful to students to solve matrix problems in the class.
- 3. Calculations on $m \times n$ matrix, (especially large *m* and *n*), will be very easily done with the help of matlab program that was been created in the m-file.
- 4. Solving the matrix problems using m-file program requires a relatively short time.

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