Literature Review: Trends of Matematical Competences and Learning Activity

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Abstract—This paper present a topic about trends of mathematical competence of some countries are ongoing, both in the education curriculum as well as in the learning process. Trends of mathematical competence are discussed more focused on several competencies related to "mathematical proficiency". Next will be discussed about how learning activities were implemented by teachers to support the establishment of competencies that will be enhanced during the learning process. Discussion of trends of competence and learning activities is a literature review on some recent articles obtained from recognized journals.

Keyword: Mathematical Competence, Learning Activity

I. INTRODUCTION

Mathematical competence is one of the cores in mathematics that will be developed and upgraded. Students who studied mathematics directed to have the skills and mathematical proficiency. Skills and mathematical proficiency will certainly be supported by mathematical competencies possessed by the student (Niss et al., 2016). The extent to which such competence has been practiced can be seen by the place, context, and level of education. In general, the design and implementation of learning plan contain competencies explicitly arranged so that in the learning process, teachers and students have more focused objectives.

Mathematical competence which has been prepared in planning future learning will be raised, developed and improved during the process of mathematical learning takes place. Learning activities created and implemented by teachers and students adjusted to the competencies that wish to grown them. The learning activities are prepared and implemented in accordance with the field and the existing situation. Some differences in learning activities can occur because of differences in priorities competency to be grown (Niss et al., 2016). Differences in priorities must be adjusted to the needs of competence in the place where the place, context and level of education continues.
II. TRENDS OF MATHEMATICAL COMPETENCE

Mathematical competence is the purpose of learning mathematics is certainly evolved over the times and the demands of scientific developments. The mathematical competence is an ability that should be owned by the students after undergoing a process of learning mathematics that followed. Mathematical competence is interrelated and mutually support each other, so that the necessary preparations and planning in drafting learning.

A lesson plan that will be implemented during the learning process is arranged in a form of learning as curriculum planning. In every country, there is a mathematical curriculum different. Preparation of different curricula is of course prepared in accordance with the needs of mathematical competencies that are applicable. Differences in mathematics lesson plan based on the mathematical competence are certainly influenced by the demands of more sophisticated competencies according to the times.

The growth and development of the mathematical competence change over time and become a unique trend. In this paper, the authors will describe the trend of mathematical competence is being developed by several countries at the moment. Trends mathematical competence is gained from the study of literature some recent articles that discuss the development of the mathematical competence in each of these countries.

A. United States

Development of education in the United States has a major impact on the development and practice of mathematics education worldwide (Niss et al., 2016). There are three groups in the USA recently produced different conceptualizations of what it means to know and use mathematics related to mathematical competence. One of the first systematic attempts to capture significant aspects of mathematical competence was made in the USA by the National Council of Teachers of Mathematics (NCTM). NCTM identified five oriented students mathematics competence: (1) learn to value mathematics, (2) become confident in their ability to do mathematics, (3) become a problem solver, (4) learn to communicate mathematically, and (5) learn to reason mathematically (Niss et al., 2016). This resulted in the publication Principles and Standard for School Mathematics (NCTM, 2000) that have six basic principles for school mathematics education: equity, curricula, teaching, learning, assessment, and technology, and put six process standard (competence): Problem Solving, Reasoning, Proof, Communication, Connections, and Representations.

A second group is National Research Council (NRC) provided a conceptualization of successful mathematics learning that called “mathematical proficiency” (Kilpatrick et al., 2001). Competence aspect of proficiency they proposed are:

- conceptual understanding – comprehension of mathematical concepts, operations and relations
- procedural fluency – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- strategic competence – ability to formulate, represent and solve mathematical problems
- adaptive reasoning – capacity for logical thought, reflection, explanation, and justification
- productive disposition – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy.

The third group is RAND Mathematical Study Panel (Boaler, 2002, Niss et al., 2016). The panel referred to these proficiencies as forming the conception of what it means to be competent in mathematics. They proposed “mathematical practices” that describe as:
this area focuses on the mathematical know-how, beyond content knowledge, that characterizes expertise in learning and using mathematics. The term 'practices' refers to specific things that successful mathematics learners and users do. Justifying claims, using symbolic notation efficiently and making generalizations are examples of mathematical practices.

In this conception, students develop knowledge and develop the relationship with the knowledge. The main focus is not the learning mathematics, but the doing of mathematics as learner and problem solvers. The mathematical competence is given such as mathematical representation, attentive use of mathematical language and definition, articulated and reasoned claims, rationally negotiated disagreement, generalizing ideas and recognizing patterns, problem-solving, and communication (Niss et.al., 2016). Examples of mathematical practices include justification, representation, reconciliation, action to making connections and seeing mathematical relations as a dynamic process (Boaler, 2002).

B. Australia and New Zealand

At Australia, mathematics is positioned to involve observing, representing and investigated pattern and relationship in social and physical phenomena and between mathematical object. It gave emphasis to mathematical product and mathematical process. Specific outcomes for working mathematics in the areas of investigating, conjecturing, using problem-solving strategies, applying and verifying, using mathematical language, and working in context. The interaction included three content strand (number and algebra, measurement and geometry, and statistics and probability) and four competence strands (understanding, fluency, problem-solving, and reasoning) (Niss, et.al., 2016).

Whilst the framework for mathematics in New Zealand is structured around three content strand, without explicit reference to mathematical competencies or process, objectives for each of the defined levels refer to thinking mathematically and statistically and to the need in each content area to solve problems and model situation (Niss, et.al., 2016).

C. Danish

At Danish, mathematical competence means to have knowledge about, to understand, to exercise, to apply, and to relate to and judge mathematics and mathematical activity in a multitude of contexts which actually do involve, or potentially might involve mathematics. These eight mathematical competencies are (Niss, et.al., 2016):

- Mathematical thinking
- Problem handling
- Modeling
- Reasoning
- Representation
- Symbols and formalism
- Communication, and
- Aids and tools

D. Germany, Austria, and Switzerland

Six general Germany mathematical competencies formed what was termed the core of mathematics standards (Niss, et.al. 2016):

- To reason mathematically
- To solve problems mathematically
- To do mathematical modeling
- To use mathematical representations
To deal with the symbolic/formal/technical aspects of mathematics
To communicate mathematically

The Austria framework specifies three dimensions, called mathematical action, mathematical content, and complexity. The dimension of mathematical action consists of four domains of action: Representing (building models), computing and operating, interpreting, and reasoning and justifying, which is a category of the same kind as the one called mathematical competencies elsewhere. The four content domains are: “numbers and measures”, “variables, functional dependencies”, “geometrical figures and solids”, and “statistical representation and descriptors”. The dimension of complexity looks at how involved the processes at issue are: “Activation of basic knowledge and skills”, “creating connections”, and “Activation of reflective knowledge, reflecting” (Niss et.al., 2016).

Then, the Swiss standards identifies eight fundamental aspects of mathematical action: knowing, realizing and describing; operating and computing; employing instruments and tools; representing and communicating; mathematising and modelling; reasoning and justifying; interpreting and reflecting on results; investigating and exploring corresponding to what elsewhere is called competencies (Niss et.al., 2016).

E. Asia

Generally, at Asia (included East Asia and South East Asia), the mathematical competence has some mathematical competence (Niss, et.al., 2016):

- Mathematical problem solving
- Mathematical reasoning,
- Connecting mathematical conceptual and procedural knowledge
- Relating and representing mathematics,
- Critical thinking,
- Creativity, and
- Ability to communicate clearly and logically in mathematics language

III. LEARNING ACTIVITY

Swedish Classrooms

Gallos Cronberg and Emanuelsson (2013) conducted research in mathematics classes in Sweden. In math class in Sweden, students are expected to plan and work on their own on different tasks independent of the other students and to a large extent independent of the teacher. Teachers usually provided a brief introduction at the beginning of the lesson and the students then work on tasks textbooks at their own pace. During the work independently, the teacher moved to interacted with each student.

When students are given a problem or task to be solved, it will show the process of learning in students who work alone. The learning process performed by the students will be directed by teachers according to the stages of problem-solving. Problem solving can be defined as the process by which an individual attempts to find a solution to a nonroutine mathematical question. Polya (Brahier, 2010) described problem solving as a four-step process: (1) understanding the problem, (2) devising a plan for finding a solution, (3) implementing the plan, and (4) looking back at the answer to ensure that it makes sense and to determine if another plan might have been more effective.

At this research, Cronberg and Emanuelsson explore patterns of interaction, math practice, and the quality of the reasoning by focusing on the participation of the students in the entire sequence of lessons, in the form of a case study of the class. Cronberg and Emanuelsson, showing the process of learning a student, Martina. Martina learns math in a classroom environment in
which students are required to plan and work alone on different tasks, independently of the
other students and to a large extent independent of the teacher.

Starting from the question Hansson (2010) who questioned the extent to which such an
environment and related pedagogical practices to support students to develop the Appropriate
level of mathematical ability, the authors examine how Martina adapt to the learning
environment, how he interacts with others in the community, in relation to the learning
outcomes associated with the practices of mathematics and mathematical reasoning. The author
also examines how Martina is able to contribute to the norms sociomathematical in the
classroom, engage in mathematical discourse associated with the explanation, justification, and
argumentation. Martina considers the book as a primary source of support, and as a result,
learning outcomes rather depend on the suitability of instructional text to help the development
of learning to the next zone.

In the learning process, it appears that each student (in this case Martina) are given a problem
to solve. Provision of such problems is the basis of learning model Problem Based Learning, PBL
is a learning model, in which the learning process begins with a problem to be solved
(Padmavathy & Mareesh, 2013). The problem to be solved is created so that students need to
acquire new knowledge prior to solve the problem. Students are not emphasized in order to
obtain the solution of the problem, but more on how students interpret the problem, gather the
information needed to solve the problem, identify possible solutions, evaluate options possible
solutions and finally took a conclusion.

PBL is a learning model by giving the problem, problem-solving strategies, more
opportunities for students to think critically, express ideas, and communicate with each other,
so that the students' ability to manage their own knowledge can help them solve a problem with
either. Johnson (2005) recommends the use of PBL as a learning model because it offers a
cognitive approach and a collaborative approach to solve, problems that are realistic, where
students are directed to resolve the problems that diiberikan independently. As well as focusing
on the students in the learning process, PBL also allows students and teachers to decide what
they need to resolve, so that students gradually take responsibility for their own learning.

Ross (Hillman, 2003) defines PBL as a result of learning gained from the process of learning to
gain an understanding based on a given problem. Fogarty (Hillman, 2003) defines PBL as a
model curriculum designed by the problems that exist in real life are arranged in ill structured,
open-ended, or ambiguous. The basic premise of PBL is that the starting point for learning is a
problem to be solved or solved by the students. Learning materials created and taught using the
issue as a motif and focus students' activities. PBL according Cazolla (2008) is an instructional
models with a student-centered approach, where students are required to do research,
integrating theory and practice, and apply the knowledge and skills to develop the right solution
to the problem given to them. In other words, learning PBL is a constructivist model learner-
centered instructional approach based on analysis, resolution and discussion on issues given to
such learners.

From the explanation above, it can be said that the PBL as a learning approach that begins
with the presentation of a problem that is designed in the context of relevant materials to be
studied to encourage students to: acquire knowledge and understanding of concepts, achieving
critical thinking, to have independent learning, skills to participate in group work, and problem
solving skills.

Sears and Hersh (Permana & Sumarmo, 2007), suggests some characteristics of PBM, namely:
(a) The problem must be related to the curriculum, (b) The problem is ill-structured problems,
the solution is not a single, and the process is gradual, (c) Students solve problems and teachers
as facilitator, (d) The students were given only a guide to identify a problem, and was not given
a formula to solve the problem, and (e) an authentic performance-based assessment. Pierce and
Jones (Dasari, 2003) PBM classified into two levels, namely the low level and high level. PBM classified as low-level if it only contains a little of the above characteristics, and PBM classified at a higher level if students are actively involved in activities that reflect the characteristics of PBM above.

With all the knowledge and abilities they already had, students are required to resolve the problem that is rich with mathematical concepts. Characteristics of PBM them are: 1) positioned students as self-directed problem solver through collaborative activities, 2) encourage students to be able to find the problem and elaborated by asking the allegations and to plan completion, 3) to facilitate students to explore various alternative settlement and implications, as well as collecting and distributing information, 4) train students to skillfully present the findings, and 5) familiarize the students to reflect on the effectiveness of their way of thinking in solving the problem.

One of the goals of PBL is to create a learning procedure that is able to identify and apply previous learning, through an approach that includes exploration of the problem, information retrieval and synthesis of information found, build derivative and of runs the problem, and solve it based on previous knowledge gained from reading (Neufeld ; Boud; in Hillman, 2003). In this way, students have the opportunity to use or process the knowledge they gained from the meeting and previous learning. In addition, students also have the opportunity to contribute so that membentu confidence in their scholastic aptitude (Hillman, 2003).

With the model of PBL in mathematics learning, students have the ability to think with creative, capable of taking decisions, and be able to solve the problem. In addition, learning by PBL models have the effect of giving greater opportunity for students to learn with the involvement of more and increase active participation by learners, as well as increased motivation to learn. This causes the students to have a positive attitude towards mathematics and helping them to improve their performance.

Before the teacher decided to use the model PBL accomplishments in learning when teaching mathematics. First teacher must have confidence that they actually use PBL in their teaching practice. Thus, the teacher will have confidence that the model PBL is one model that fits and is good for use. In other words, their personal experience in using the model PBL is the best basis for them to be able to build effective teaching (Cazzola, 2008).

PBL learning process shaped cycle, which has three phases, where each phase containing characteristic of PBL (Johnson, 2005). The third phase is described as follows:

1. Phase of development issues, has the characteristics of PBL, namely, #1. The formation of a group that would solve the problem
   Forming the group is aiming to understand each other and get to know what skills each member will be involved in solving the problem, so based on the potential of expertise of each member of the group can be prepared several plans and measures the contribution of each member. Thus, the learning process has a comfortable climate for collaborative learning and expected to give each other feedback that supports one another.

2. Phase troubleshooting process, has the characteristics of PBL are #2. Identifying problems
   The task group in this phase is to evaluate and identify different aspects of the problem and obtain information about the underlying issues. Identification and understanding of the issues can be directed to the questions: What do we think about this issue? What we will do? What do we already know? What else do we need to know?
   #3. Working to solve the problem by involving every member.
Process of problem solving is done by involving every member of the group, by using and applying the skills, abilities, and knowledge of each member of the group. The problem solving process can be prepared and translated into stages to obtain an acceptable solution and believed to be true by every member of the group.

#4. Dividing partition problems to be done by each member

Each member of the group to identify what is known and what is unknown to solve the problem. Each member has the task to investigate the issue further. Each member can go to the lab, to the library, read articles and so forth. Then each member came back with the information and knowledge acquired respectively to the group as part of the solutions offered to help solve the problem.

3. Solution problem

#5. Evaluate and implement the work of members.

Each member is required to share what they have learned, to reconsider their hypothesis, or to generate new hypotheses corresponding new understanding that they get in new learning experiences.

#6. Make the conclusion of the solution.

The findings from each of the small groups then reported and discussed in larger groups. So in the end the problem can be resolved. The knowledge that has been owned integrated as the new knowledge and skills that can be the basis for use in the next learning.

Problems are given to students is ill structured, where the teacher acts as a coach and facilitator independent study on the issue. In PBL, the teacher provides information that can provide opportunities for students to learn independently. Teachers also support and guide student learning through instruction scaffolding to help manage the learning task. Furthermore, students are encouraged to use metacognition in learning so that teachers can assess students' learning difficulties, provide evaluation feedback. Therefore, the scaffolding is very important for students who do not prepare well within the ability of strategic thinking. Scaffolding is also an element that sets it apart from learning PBL invention. In PBL, students are given a problem to solve. Training the students are not the same as giving them problems. But this does not mean that because the issue has been given to the students will also be taught problem solving. Problem solving is not necessarily enhanced by the use of PBL (Hillman, 2003).

In this research, seen that Martina comfortable interacting with either the teacher or classmates. Martina is able to work with classmates to discuss the solution of mathematical problems. In addition, from the intensity of participation is given in the classroom shows the level of mathematical reasoning such as using equations and graphs for a particular topic, although sometimes he tries to use a calculator or ask the help of teachers. Martina development quality mathematical reasoning can be seen from the ability to choose and use a conceptual approach to solve mathematical tasks and find solutions to problems. Martina is able to participate in a learning environment using the tools of mathematics and mathematical symbols to reason and communicate mathematical ideas. And learning activity such as that independent work as a mode of instruction exists in Sweden.

New Zealand Classrooms

Anthony (2013) explore the spaces by means of students' voices illustrates how the development of mathematical proficiency cannot be separated from the axes of social and material advantage. These space filled by the student's perceptions of the "good" teacher and "good" learner in New Zealand classrooms. He explores how's students' perceptions about
good teaching relate to their views about what good learning and being a good mathematics learner look like.

He interviewed the students’ expressed satisfaction with their teacher. Students were able to articulate different teacher activities to emphasize their teachers’ effectiveness. As a result, he found some different students’ perceptions:

- **She is really helpful and she teaches me new things and she explains it really well**
  Teachers are expected to understand the mathematical abilities of students, create equal opportunities for all students to contribute, train and shape students to be confident and competent in mathematics. Teachers should be make sure that students get the task and the right opportunities, so students have to study the progress of mathematics and mathematical can work efficiently. In explaining the lesson, the teacher is expected to give a clear explanation; teachers help students see the connection between concrete ways and formal in solving mathematical problems. Students liked the records provided every lesson where the teacher explains what is needed and what they should do on every issue. Students expect teachers to make them into students’ someone good number ”,” someone who can find alternative ways to work out problems ”,” someone who is doing their homework ”. Teachers are expected to become a man of preparation for listen and help students, uncooperative and did not make the gap between teachers and students. Sometimes students want teachers to explain and demonstrate how to do a task math by showing how teachers doing math.

- Some students assessed their teacher as ‘good’ along two dimensions: his actions associated with care and on the ways in which he made mathematics accessible. Collectively, these students offered a range of pedagogic actions that endorsed the teacher’s efforts to create an environment that was responsive to the diverse sociopolitical realities of the class.
  Students appreciate and appreciate teachers who understand them as individuals, tolerated their need to express themselves, encouraging them to participate, and boost their confidence as learners of mathematics. Teachers care for students, including respect for the individual as a social person and student of mathematics. Teachers are also expected to know the personality of their students. In the class expected role of teacher as explainer. Students want the teacher’s explanations make things complicated math becomes simple and easy for them to follow. Creating a complex mathematical topic is broken down into several parts that are simpler and easier to understand. In response to the explanation given by the teacher, students often feel that their role as a good student is to "listen to the teacher."

- Other students believed that their teacher was ‘good’ because she helped students to understand the mathematics, understanding was associated with the teachers’ role as a ‘good’ explainer. In relation to explaining, the students like that the teacher encouraged them to communicate their mathematical thinking. They valued that the teacher appeared to not become frustrated nor expect his students to immediately understand mathematical concepts or know how to solve all problems.
  Students interpret their teacher’s concerns against the desire to learn and understand mathematics. This can be demonstrated in ways that teachers form a community or group learning, where students can make their own decisions on what he learned. In taking the decision, students are required to communicate is not just about what they know, but also what they do not understand. Students
appreciate the teacher as explainer, not because his explanation makes math problems easy to solve, but as an explanation challenge and deepen their understanding of mathematics; supporting the formation of students' conceptual understanding. Teachers engage students in every step of solving problems, not solve the problem by themselves. While some expressed the need to know a lot of basic maths and work hard, several students were explicit in the need to communicate with the teacher, and to seek help to understand. However, what was common across all student responses was the need to be prepared to struggle with the mathematics, offering their own ideas, and attending to the mathematical thinking of others in the class.

What was also apparent in this analysis is that ‘good’ teachers and their ‘good’ learners construct unique learning communities. Each class learning environment comprised significantly different activities and associated mathematical practices that variously afforded or constrained students’ opportunities to develop mathematical proficiency. Understanding how students and teachers construct notions of ‘good’ practices of teaching and learning within the classroom offers possibilities of deepening our understanding of how students and teachers contribute to the ongoing regeneration of the normative identity as doers of mathematics. Understanding how mathematical identities evolve and how students develop a sense of affiliation with mathematical activity as it is realized in their classroom must continue to be of primary concern to contribute to the improvement of equitable and culturally responsive learning and teaching.

Norwegian 9th Grade Classrooms

Bergem and Pepin (2013) analyzed the dialogue between teachers and students in 9th grade classrooms of the Norwegian. They analyzed how the dialogue between teachers and students can develop students mathematical proficient based on mathematical problems set by the teacher. In their study the teacher’s views on how to develop student learning of “equation” guided her use of specific instructional strategies. These included “simple” word examples as introductory activity, particular representations to “picturise” equations, and her ways of setting an “equation” question out on the board. They contend that these selected strategies are likely to develop mathematical competence or proficiency.

That appropriate mathematical tasks are those that make the mathematics “problematic” for pupils; problematic in the sense that pupils regard the task as an interesting problem, for them, something worth finding out, “something to make sense” to them. But in fact, the selected tasks did not seem to offer pupils sufficient opportunities to reflect and communicate; these tasks were too easy (in terms of finding the solutions), and they were not genuine authentic problems for these learners. In the present classroom many students reacted to the non-challenging task with raised voices and arguments indicating that they did not find learning about equations to be “sensible, useful and worthwhile”. However, developing an inclination to see mathematics as sensible, useful and worthwhile is an important part of mathematical proficiency, as described by Kilpatrick et al. (2001).

This is also documented in their study, that students could agree or refuse to come to the board when asked by the teacher; they could probe for deeper understandings (“what is the known”); or simply answer the teachers’ questions. But, creating learning communities within the mathematics classroom, based on democratic participation and mathematical proficiency, requires that other strands of mathematical proficiency come into action. It believes that the effectiveness of teaching and learning situation depends on several factors: students who are actually involved; curriculum materials and assignments; and also on the various decisions taken by the teacher during a math lesson.
IV. CONCLUSION

Mathematical competence is very important and a target goal of learning mathematics. Trends learning of mathematics is still centered on the development of mathematical cognitive competencies such as mathematical proficiency. The mathematics learning tailored to the mathematical competence that will be grown and developed in each place. Differences in priorities must be adjusted to the needs of competence in the place where the place, context and level of education continues.

In addition to the trend of mathematical competence is still developing in the form of mathematical proficiency competence (mathematical proficiency) learning trend also leads to the learning model of problem solving. It appears from some of the descriptions of learning activities that show that learning begins with the provision of a mathematical problem to cultivate students' mathematical abilities. With the various observations made by several researchers, either by conducting interviews to students, as well as observing the learning process that takes place, it appears that the trend is more focused on problem-based learning models.

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